

Sherlock Holmes, Mathematician

“Crime is common. Logic is rare. Therefore it is upon the logic rather than upon the crime that you should dwell.”

– “The Adventure of the Copper Beeches” [CSH, p. 317]

Holmes on the Method of Reasoning Backwards from Effects to Causes

"In solving a problem of this sort, the grand thing is to be able to reason backward. That is a very useful accomplishment, and a very easy one, but people do not practise it much. ... Most people, if you describe a train of events to them, will tell you what the result would be. They can put those events together in their minds, and argue from them that something will come to pass. There are few people, however, who, if you told them a result, would be able to evolve from their own inner consciousness what the steps were which led up to that result. This power is what I mean when I talk of reasoning backward, or analytically."

A Study in Scarlet, Part 2, ch. 7 [CSH, pp. 83-4]

The Role of Probabilities in Holmes's Method

"We are coming now rather into the region of guesswork," said Dr. Mortimer.

"Say rather, into the region where we balance probabilities and choose the most likely. It is the scientific use of the imagination, but we have always some material basis on which to start our speculation. Now, you would call it a guess, no doubt, but I am almost certain that this address has been written in a hotel."

"How in the world can you say that?"

"If you examine it carefully you will see that both the pen and the ink have given the writer trouble. The pen has spluttered twice in a single word and has run dry three times in a short address, showing that there was very little ink in the bottle. Now, a private pen or ink-bottle is seldom allowed to be in such a state, and the combination of the two must be quite rare. But you know the hotel ink and the hotel pen, where it is rare to get anything else."

The Hound of the Baskervilles, ch. 4 [CSH p. 687]

The Use of Statistical Data

"There was no wound upon the dead man's person, but the agitated expression upon his face assured me that he had foreseen his fate before it came upon him. Men who die from heart disease, or any sudden natural cause, never by any chance exhibit agitation upon their features. Having sniffed the dead man's lips, I detected a slightly sour smell, and I came to the conclusion that he had had poison forced upon him. Again, I argued that it had been forced upon him from the hatred and fear expressed upon his face. By the method of exclusion, I had arrived at this result, for no other hypothesis would meet the facts. Do not imagine that it was a very unheard-of idea. The forcible administration of poison is by no means a new thing in criminal annals. The cases of Dolsky in Odessa, and of Leturier in Montpellier, will occur at once to any toxicologist."

A Study in Scarlet, ch. 7 [CSH, p. 84]

The Method in Action: The Case of the Watch

"I have heard you say it is difficult for a man to have any object in daily use without leaving the impress of his individuality upon it in such a way that a trained observer might read it. Now I have here a watch which has recently come into my possession. Would you have the kindness to let me have an opinion upon the character or habits of the late owner?"

I handed him over the watch with some slight feeling of amusement in my heart, for the test was, as I thought, an impossible one, and I intended it as a lesson against the somewhat dogmatic tone which he occasionally assumed. He balanced the watch in his hand, gazed hard at the dial, opened the back, and examined the works, first with his naked eyes and then with a powerful convex lens. I could hardly keep from smiling at his crestfallen face when he finally snapped the case to and handed it back.

"There are hardly any data," he remarked. "The watch has been recently cleaned, which robs me of my most suggestive facts."

"You are right," I answered. "It was cleaned before being sent to me."

In my heart I accused my companion of putting forward a most lame and impotent excuse to cover his failure. What data could he expect from an uncleaned watch?

"Though unsatisfactory, my research has not been entirely barren," he observed, staring up at the ceiling with dreamy, lacklustre eyes. "Subject to your correction, I should judge that the watch belonged to your elder brother, who inherited it from your father."

"That you gather, no doubt, from the H. W. upon the back?"

"Quite so. The W. suggests your own name. The date of the watch is nearly fifty years back, and the initials are as old as the watch: so it was made for the last generation. Jewellery usually descends to the eldest son, and he is most likely to have the same name as the father. Your father has, if I remember right, been dead many years. It has, therefore, been in the hands of your eldest brother."

"Right, so far," said I. "Anything else?"

"He was a man of untidy habits--very untidy and careless. He was left with good prospects, but he threw away his chances, lived for some time in poverty with occasional short intervals of prosperity, and finally, taking to drink, he died. That is all I can gather."

I sprang from my chair and limped impatiently about the room with considerable bitterness in my heart.

"This is unworthy of you, Holmes," I said. "I could not have believed that you would have descended to this. You have made inquiries into the history of my unhappy brother, and you now pretend to deduce this knowledge in some fanciful way. You cannot expect me to believe that you have read all this from his old watch! It is unkind and, to speak plainly, has a touch of charlatanism in it."

"My dear doctor," said he kindly, "Pray accept my apologies. Viewing the matter as an abstract problem, I had forgotten how personal and painful a thing it might be to you. I assure you, however, that I never even knew that you had a brother until you handed me the watch."

"Then how in the name of all that is wonderful did you get these facts? They are absolutely correct in every particular."

"Ah, that is good luck. I could only say what was the balance of probability. I did not at all expect to be so accurate."

"But it was not mere guesswork?"

"No, no: I never guess. It is a shocking habit--destructive to the logical faculty. What seems strange to you is only so because you do not follow my train of thought or observe the small facts upon which large inferences may depend. For example, I began by stating that your brother was careless. When you observe the lower part of that watch-case you notice that it is not only dented in two places but it is cut and marked all over from the habit of keeping other hard objects, such as coins or keys, in the same pocket. Surely it is no great feat to assume that a man who treats a fifty-guinea watch so cavalierly must be a careless man. Neither is it a very far-fetched inference that a man who inherits one article of such value is pretty well provided for in other respects."

I nodded to show that I followed his reasoning.

"It is very customary for pawnbrokers in England, when they take a watch, to scratch the numbers of the ticket with a pin-point upon the inside of the case. It is more handy than a label as there is no risk of the number being lost or transposed. There are no less than four such numbers visible to my lens on the inside of this case. Inference--that your brother was often at low water. Secondary inference--that he had occasional bursts of prosperity, or he could not have redeemed the pledge. Finally, I ask you to look at that inner plate, which contains the keyhole. Look at the thousands of scratches all round the hole--marks where the key has slipped. What sober man's key could have scored those grooves? But you will never see a drunkard's watch without them. He winds it at night, and he leaves these traces of his unsteady hand. Where is the mystery in all this?"

The Sign of Four, ch. 1 [CSH, pp. 92-3]

Dr. Joseph Bell: The Man Behind the Legend

In 1878 Sir Arthur Conan Doyle, then a young medical student, was selected to be the outpatient clerk for Dr. Joseph Bell, a famous Edinburgh physician. It is widely known that Bell served as the model for Sherlock Holmes, at least insofar as “reasoning backwards, or analytically” was concerned.

What is not so widely known is that in that very year the notorious murderer Eugene Chantrelle was hanged in Edinburgh for the murder of his wife. As he ascended the gallows, Chantrelle singled out the man who had solved the crime and brought him to justice: Dr. Joseph Bell.

Bell always insisted that his work “for the crown” was not for public discussion and he gave, in public at least, no information regarding the role he played in this case. But Conan Doyle pressed Bell in private communication for illustrations of the method of reasoning from effects to causes that might provide material for his stories. It is probable that we owe some of the instances of Holmes’s reasoning to the materials supplied by Bell.



In a letter dated May 4, 1892, Conan Doyle wrote to Bell:

“It is most certainly to you that I owe Sherlock Holmes, and though in the stories I have the advantage of being able to place [the detective] in all sorts of dramatic positions, I do not think that his analytical work is in the least an exaggeration of some of the effects which I have seen you produce in the outpatient ward. Round the centre of deduction and inference and observation which I have heard you inculcate, I have tried to build up a man who pushed the thing as far as it could go – further occasionally – and I am so glad that the results satisfy you, who are the critic with the most right to be severe.”

What were those effects? Conan Doyle writes of one of them in which Bell interviews a civilian patient:

“Well, my man, you’ve served in the army.” “Aye, sir.”

“Not long discharged?” “No, sir.”

“A highland regiment?” “Aye, sir.”

“A non-com. officer?” “Aye, sir.”

“Stationed at Barbados?” “Aye, sir.”

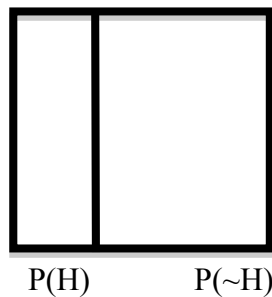
“You see, gentlemen,” he would explain, “the man was a respectful man, but did not remove his hat. They do not in the army, but he would have learned civilian ways had he been long discharged. He has an air of authority and he is obviously Scottish. As to Barbados, his complaint is elephantiasis, which is West Indian and not British.”

To his audience of Watsons it all seemed quite miraculous until it was explained, and then it became simple enough. It is no wonder that after a study of such a character I used and amplified his methods when in later life I tried to build up a scientific detective who solved cases on his own merits and not through the folly of the criminal.

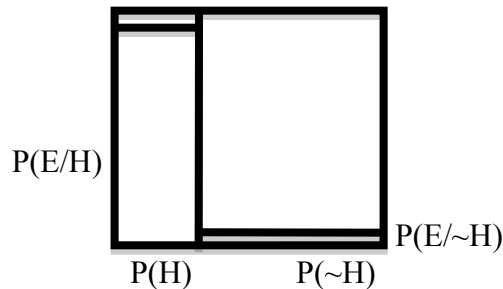
The Mathematical Basis of Holmes's Method

To understand Holmes's method, we need to give a rational reconstruction of that method – to reveal its structure in a way that enables us to demonstrate that it is indeed reasonable to make such inferences. Fortunately, it is not difficult to do this using the mathematical tools of Bayesian probability theory.

Take the case of the hotel pen from *The Hound of the Baskervilles*. Holmes tacitly divides up the possible writing implements (remember that these are fountain pens, not ball point pens!) into the privately-owned and the publically-owned, the latter being those available in hotels. Though he does not indicate an explicit proportion between them, we may infer from the line of his reasoning that he knew hotel pens to be not vastly less numerous than private pens. We can use this as a baseline for a prior probability, perhaps assigning the numbers $P(H) = .3$ and $P(\sim H) = .7$ as the probabilities that the pen in question was a hotel pen or not a hotel pen, respectively. These can then be incorporated into a square diagram:



What brings the inference around firmly to the side of its being a hotel pen is the evident trouble it has given the writer, sputtering and running dry, which reveals that the ink bottle into which the pen must be repeatedly dipped had run nearly dry. Private owners of pens, Holmes remarks, no doubt relying on his experience again, rarely permit them to be in such a state: but in a hotel where no user has a motivation to take thought for the next, “it is rare to get anything else.” The poor state of the pen, then, has different conditional probabilities on the two available hypotheses. We might emulate Holmes's judgements with the probability assignments $P(E/H) = .95$ and $P(E/\sim H) = .03$. These respective likelihoods, as they are called, are plotted orthogonally to the prior probabilities:



The probability of the hotel pen hypothesis H given the evidence is now the proportion of the shaded region on the left to the total shaded region. Bayes's Theorem gives us the expression for $P(H/E)$ (below left), and simple algebra (below right) confirms what the geometry makes evident to the eye: H is powerfully confirmed by E .

$$\frac{P(H) \times P(E/H)}{P(H) \times P(E/H) + (P(\sim H) \times P(E/\sim H))}$$

$$\frac{.3 \times .95}{(.3 \times .95) + (.7 \times .03)}$$

This works out to over 93%, even though initially the odds were more than 2 to 1 against a hotel pen.