

Symbolic Logic — Rules of Transformation in Lemmon’s Proof System — Jonah N. Schubach

<p><i>Rule of Assumptions (A)</i> [p. 9]</p>	<p>Any wff ϕ may be introduced at any stage of a proof given itself as an assumption of the proof. ** Assumption = wff included in a proof but not derived <i>In the Proof System:</i> we write the wff down on a new line, write A to the right of it, and put its own line number to the left of it (where we keep track of assumptions)</p>
<p><i>Modus (Ponendo) Ponens (MPP)</i> [p. 9]</p>	<p>Given ϕ and $(\phi \rightarrow \gamma)$, we may derive γ. Conclusion γ depends on any assumptions on which ϕ or $(\phi \rightarrow \gamma)$ depends. <i>In the Proof System:</i> we write the wff γ down on a new line, write “MPP” to the right of it along with the line numbers corresponding to wffs ϕ and $(\phi \rightarrow \gamma)$. To the left, we write down all of the assumptions that ϕ or $(\phi \rightarrow \gamma)$ depends on.</p>
<p><i>Modus (Tollendo) Tollens (MTT)</i> [p. 12]</p>	<p>Given $\neg\gamma$ and $(\phi \rightarrow \gamma)$, we may derive $\neg\phi$. Conclusion $\neg\phi$ depends on any assumptions on which $\neg\gamma$ or $(\phi \rightarrow \gamma)$ depends.</p>
<p><i>Double Negation (DN)</i> [p. 13]</p>	<p>Given wff ϕ, we may derive $\neg\neg\phi$. Given wff $\neg\neg\phi$, we may derive ϕ. In either case, the conclusion depends on the same assumptions as the premise.</p>
<p><i>Conditional Proof (CP)</i> [p. 14]</p>	<p>Given a proof of γ resting upon ϕ as an assumption, we may derive $(\phi \rightarrow \gamma)$ on the remaining assumptions (if any).</p>
<p><i>&-Introduction (&I)</i> [p. 19]</p>	<p>Given ϕ and γ, we may derive $(\phi \& \gamma)$. Conclusion $(\phi \& \gamma)$ depends on any assumptions on which ϕ or γ depends.</p>
<p><i>&-Elimination (&E)</i> [p. 20]</p>	<p>Given $(\phi \& \gamma)$, we may derive either ϕ or γ separately. In either case, the conclusion depends on the same assumptions as the premise.</p>
<p><i>\vee-Introduction (\veeI)</i> [p. 22]</p>	<p>Given either ϕ or γ separately, we may derive $(\phi \vee \gamma)$. In either case, the conclusion depends on the same assumptions as the premise.</p>
<p><i>\vee-Elimination (\veeE)</i> [p. 22]</p>	<p>Given $(\phi \vee \gamma)$, together with a proof of λ resting upon ϕ as an assumption and a proof of λ resting upon γ as an assumption, we may derive λ. Conclusion λ depends on any assumptions on which $(\phi \vee \gamma)$ depends plus those on which λ depends in its derivation from ϕ (apart from ϕ), plus those on which λ depends in its derivation from γ, (apart from γ).</p>
<p><i>Reductio ad Absurdum (RAA)</i> [p. 26]</p>	<p>Given a proof of $(\gamma \& \neg\gamma)$ resting upon ϕ as an assumption, we may derive $\neg\phi$ as a conclusion resting on the remaining assumptions (if any).</p>